Math 290 ELEMENTARY LINEAR ALGEBRA PRACTICE FINAL (for In-Class)

December 9 (Wed), 2017

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Line #: 25751.

ID # : <u>Name</u> :

 \star The actual exam may not be very similar to this practice exam. The purpose of this practice exam is to give you an idea of how the actual exam will look like, in terms of the length and the format. This practice exam is for the "in-class" portion of the exam only.

[I] (20pts)

(1)
$$\det \begin{bmatrix} 4 & 0 & 1 \\ 3 & 0 & 2 \\ 2 & 0 & 3 \end{bmatrix} = \underline{\qquad}.$$

(2)
$$\det \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix} = __.$$

(3)
$$\det \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} = \underbrace{1}_{1}$$

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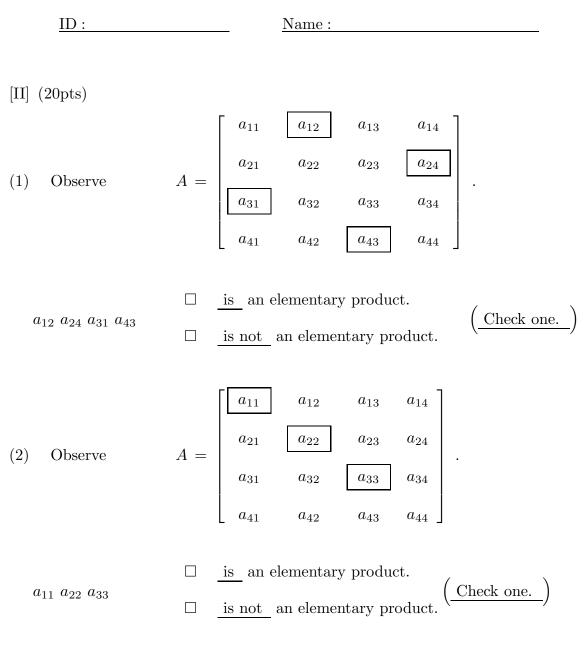
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([I] continued)

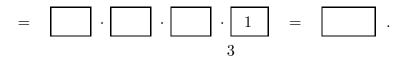
(4)
$$\begin{vmatrix} 1 \cdot 1 & 1 \cdot 2 & 1 \cdot 3 \\ 2 \cdot 1 & 2 \cdot 2 & 2 \cdot 3 \\ 3 \cdot 1 & 3 \cdot 2 & 3 \cdot 3 \end{vmatrix} = __.$$

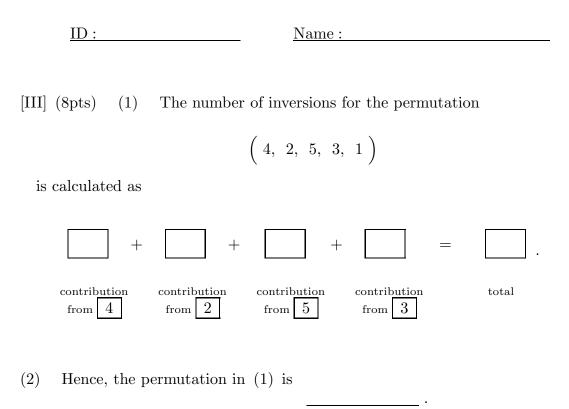
(5)
$$\begin{vmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{vmatrix} = __.$$

(6)
$$\begin{vmatrix} 1 & -1 & -1 & -1 \\ 1 & 1 & 1 & -1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 \end{vmatrix} = \underline{\qquad}.$$



(3) The number of elementary products of the general 4×4 matrix A





 $\left(\underline{ \ \ Fill \ in \ either \ "even" \ or \ "odd" \ } \right)$

(3) In the formula defining the determiant

| | a_{11} | a_{12} | a_{13} | a_{14} | a_{15} | |
|--------|----------|----------|----------|----------|---|---|
| | a_{21} | a_{22} | a_{23} | a_{24} | a_{25} | |
| \det | a_{31} | a_{32} | a_{33} | a_{34} | a_{35} | , |
| | a_{41} | a_{42} | a_{43} | a_{44} | a_{45} | |
| | a_{51} | a_{52} | a_{53} | a_{54} | $egin{array}{c} a_{15} \\ a_{25} \\ a_{35} \\ a_{45} \\ a_{55} \end{array}$ | |

 $\Box + a_{14} a_{22} a_{35} a_{43} a_{51}$ $\Box - a_{14} a_{22} a_{35} a_{43} a_{51}$ 4appears as a term. (<u>Check one.</u>)

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[IV] (12pts)

$$(1) \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^{2} & b^{2} & c^{2} \end{vmatrix}$$

$$= 1 \cdot \begin{vmatrix} b \\ b^{2} & c^{2} \end{vmatrix} - a \cdot \begin{vmatrix} 1 & 1 \\ b^{2} & c^{2} \end{vmatrix} + a^{2} \cdot \begin{vmatrix} 1 & 1 \\ b & c \end{vmatrix}$$
(co-factor expansion)
$$= 1 \cdot (\boxed{} - \boxed{})$$

$$- a \cdot (\boxed{} - \boxed{})$$

$$+ a^{2} \cdot (\boxed{} - \boxed{})$$

$$+ a^{2} \cdot (\boxed{} - \boxed{})$$

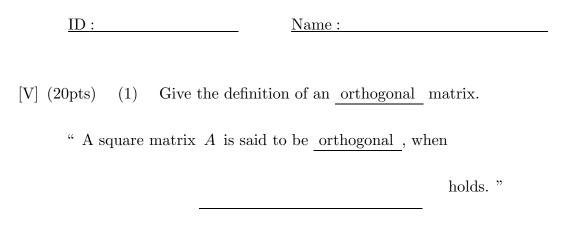
$$= bc \cdot (\boxed{} - \boxed{}) - a(c+b)(\boxed{} - \boxed{})$$

$$+ a^{2} (\boxed{} - \boxed{})$$

$$= (bc - (c+b)a + a^{2}](\boxed{} - \boxed{})$$

$$= (\boxed{b} - \boxed{a})(\boxed{} - \boxed{}) (\boxed{} - \boxed{})$$

$$(2) \qquad \begin{vmatrix} 1 & 1 & 1 \\ 1 & 4 & 9 \\ 1 & 16 & 81 \end{vmatrix} = _$$



(2) For an arbitrary orthogonal matrix A (with real numbers as entries),

(3) If
$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$
 is orthogonal, and $ad - bc = -1$, then
$$A^2 =$$
.

Proof for (3): A is written as $A = \begin{bmatrix} a & b \\ b & -a \end{bmatrix}$, with $a^2 + b^2 = 1$.

Accordingly,

$$A^{2} = \begin{bmatrix} a & b \\ b & -a \end{bmatrix} \begin{bmatrix} a & b \\ b & -a \end{bmatrix} = \begin{bmatrix} \hline & & \\ \hline & & & \\ \hline & & & \\ \end{bmatrix}$$
$$= \begin{bmatrix} \hline & & \\ \hline & & \\ \hline & & \\ \end{bmatrix} \qquad \begin{bmatrix} by & a^{2} + b^{2} & = 1. \end{bmatrix}$$

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|------|---|---|---|--|--|--|
| | | | | | | |
| [VI] | (20pts) Let $A =$ | $\begin{bmatrix} a & b \\ c & d \end{bmatrix}.$ | (1) <u>True or false</u> : | | | |
| (1a) | $A + A^T$ is a symmetric | matrix. | | | | |
| | \Box True. | \Box False | e. $\left(\underline{\text{Check one.}}\right)$ | | | |
| (1b) | AA^T is a symmetric matrix | trix. | | | | |
| | \Box True. | \Box False | e. $\left(\underline{\text{Check one.}}\right)$ | | | |
| (1c) | $AA^T = A^T A.$ | | | | | |
| | \Box True. | \Box False | e. $\left(\underline{\text{Check one.}}\right)$ | | | |
| (2) | Let | | | | | |
| | $A = egin{bmatrix} a & b \ b & d \end{bmatrix}$. | | | | | |

Agree that this is a symmetric matrix. Suppose a, b, c and d are real numbers. True or false :

"A is diagonalized by an orthogonal matrix. Namely, there exists an orthogonal matrix Q such that $Q^{-1}AQ$ is a diagonal matrix."

| True. | False. | $\left(\underline{\text{Check one.}}\right)$ |
|-------|--------|--|
| | | |

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[VII] (20pts) Diagonalize the given matrix A. If not feasible, then say 'not feasible'.

$$A = \begin{bmatrix} 3 & 5\\ 1 & 2 \end{bmatrix}.$$