## Math 290 ELEMENTARY LINEAR ALGEBRA FINAL EXAM (Take-home)

December 6 (Wed), 2017 Due date: December 15 (Fri), 2017

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Line #: 25751.

<u>ID # :</u> <u>Name :</u>

This take-home part of Midterm Exam is worth 140 points and is due in class Friday, December 15th, 2017. Submission after 1:30 pm, December 15th, will not be accepted.

[I] (Take-home; 20pts) Let A be an  $n \times n$  matrix, with entires in  $\mathbb{C}$ . Let  $\lambda = \lambda_0 \in \mathbb{C}$  be one of the eigenvalues of A. Recall that the eigenspace of A with respect to its eigenvalue  $\lambda = \lambda_0$  is defined as

$$V_{\lambda_0} = \left\{ \left. \boldsymbol{x} \in \mathbb{C}^n \right| A \boldsymbol{x} = \lambda_0 \boldsymbol{x} \right\}.$$

(1) <u>True or false</u>: " $\boldsymbol{x} \in V_{\lambda_0}, \ \boldsymbol{y} \in V_{\lambda_0} \implies \boldsymbol{x} + \boldsymbol{y} \in V_{\lambda_0}.$ "

$$\Box$$
 True.  $\Box$  False. (Check one.)

(2) True or false: "
$$\boldsymbol{x} \in V_{\lambda_0}, \ \alpha \in \mathbb{C} \implies \alpha \, \boldsymbol{x} \in V_{\lambda_0}.$$
"  
 $\Box$  True.  $\Box$  False. (Check one.

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[II] (Take-home; 30pts) Complete the field axioms.

- Field axioms. Each of  $k = \mathbb{R}$  and  $k = \mathbb{C}$  satisfies the following axioms:
  - (i)  $\alpha + \beta = \beta + \alpha$ .

(ii) 
$$\alpha + (\beta + \gamma) = ( ) +$$

\_ .

(iii) 
$$\alpha + 0 =$$

(iv) 
$$\alpha + (-\alpha) =$$
\_\_\_\_\_.

(v) 
$$\alpha\beta = \beta\alpha$$
.

(vi) 
$$\alpha(\beta\gamma) = ($$
).

(vii) 
$$(\alpha + \beta)\gamma = +$$

(viii)  $\alpha \cdot 1 =$  \_\_\_\_\_.

(ix) For 
$$\alpha \neq 0$$
, there is  $\alpha^{-1}$  such that  $\alpha \alpha^{-1} = 1$ .

 $(x) \qquad 0 \neq 1.$ 

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[III]	(Take-home; 30pts) Let	et $k$ be a field.
(1)	True or false : "In a	a field $k, 0 \cdot \alpha = 0.$ "
	$\Box$ True.	$\Box$ False. ( <u>Check one.</u> )
(2)	Give the definition of $2 \in k$ .	
	2	=
(3)	True or false : "In a "	a field $k, 1+1 \neq 0.$ "

- $\Box$  Always true, no matter what the field k is.
- $\Box$  Not always true. It depends on the field k.

$$\left( \underline{\text{Check one.}} \right)$$

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[IV] (Take-home; 30pts) Complete the vector space field axioms.

## • Vector space axioms.

Let k be a field. V is said to be a <u>k-vector space</u>, if  $\boldsymbol{x} + \boldsymbol{y} \in V$   $(\boldsymbol{x}, \boldsymbol{y} \in V)$ and  $\alpha \boldsymbol{x} \in V$   $(\alpha \in k \text{ and } \boldsymbol{x} \in V)$  are both defined, and furthermore, V has a distinguishable element **0**, such that the following (i) through (viii) are satisfied:

(i) 
$$\boldsymbol{x} + \boldsymbol{y} = \boldsymbol{y} + \boldsymbol{x}$$

(ii) 
$$\boldsymbol{x} + (\boldsymbol{y} + \boldsymbol{z}) = ($$
  $) +$ 

(iii) 
$$\boldsymbol{x} + \boldsymbol{0} =$$

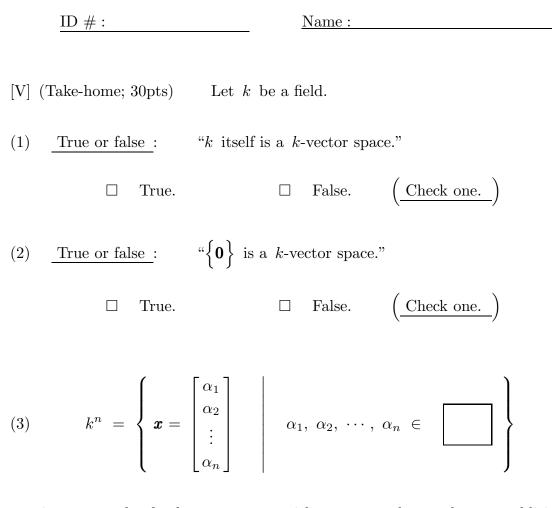
(iv) For each  $\boldsymbol{x} \in V$ , there exists  $-\boldsymbol{x} \in V$  such that  $\boldsymbol{x} + (-\boldsymbol{x}) =$ \_\_\_\_\_.

(v) 
$$\alpha(\beta \boldsymbol{x}) = ($$
  $)$ 

(vi) 
$$\alpha (\boldsymbol{x} + \boldsymbol{y}) = +$$

(vii) 
$$(\alpha + \beta)\boldsymbol{x} = +$$

(viii) 
$$1 \cdot \boldsymbol{x} =$$
 \_\_\_\_\_.



is an example of a k-vector space, with respect to the usual vector addition and scalar multiplication. (Fill an appropriate symbol in the box.)