Math 290 ELEMENTARY LINEAR ALGEBRA MIDTERM EXAM (In-Class)

October 9 (Mon), 2017

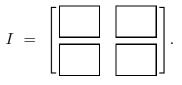
Instructor: Yasuyuki Kachi

Line #: 25751.

ID # : <u>Name</u> :

This in-class exam is worth 120 points. The duration of this exam is 50 minutes (start at 11:00am, finish at 11:50am).

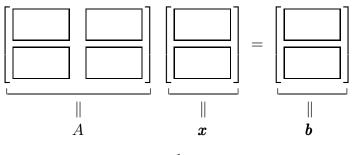
[I] (20pts) (1) What is the 2×2 identity matrix *I*? Write it out as in



(2) The system

$$\begin{cases} 2x - 7y = 5, \\ x - 6y = 1 \end{cases}$$

is rewritten as



Name : ID # : [II](20pts) Let $A = \begin{bmatrix} 5 & 2\sqrt{6} \\ 2\sqrt{6} & 5 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 9 & 4\sqrt{5} \\ 4\sqrt{5} & 9 \end{bmatrix}.$ $\det A = \underline{\qquad}.$ (1a) $\det B = \underline{\qquad}.$ (1b) $\det\left(AB\right) = \underline{\qquad}.$ (2)<u>True or false</u>: $\det (AB) = (\det A)(\det B).$ (3) \Box False. (<u>Check one.</u>) True.

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[III]	(40pts) Let $A = \begin{bmatrix} a \\ c \end{bmatrix}$	$\begin{bmatrix} b \\ d \end{bmatrix}$,	$B = \begin{bmatrix} p \\ r \end{bmatrix}$	$\begin{bmatrix} q \\ s \end{bmatrix}$. Suppose $AB = I$.			
(1)	<u>True or false</u> : $BA = I$.						
	\Box True.		False.	$\left(\underline{\text{Check one.}}\right)$			
(2)	True or false : $B = A^{-1}$						
	\Box True.		False.	$\left(\underline{\text{Check one.}}\right)$			
(3)	<u>True or false</u> : $A = B^{-1}$	•					
	\Box True.		False.	$\left(\underline{\text{Check one.}}\right)$			
(4) Explain why you chose the answers you chose for (1) , (2) and (3) above.							
Rea	son: If $AB = I$, then by			rule, $\det A$			
So	exists. Multiply		from	the to the two sides			
of	and get B	$C = A^{-}$	⁻¹ , thanks	to law.			

Then BA = I. Hence also $A = B^{-1}$. 3

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[IV] (10pts) True or False :

For

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \quad \text{and} \quad B = \begin{bmatrix} p & q \\ r & s \end{bmatrix},$$

(1)
$$(A+B)(A-B) = A^2 - B^2.$$

 \Box True. \Box False. (<u>Check one.</u>)

(2)
$$\left(\det A\right) + \left(\det B\right) = \det\left(A + B\right).$$

 \Box True. \Box False. (<u>Check one.</u>)

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[V] (10pts) Find the determinant of

$$A = \begin{bmatrix} -a^2 & ab & ac \\ ab & -b^2 & bc \\ ac & bc & -c^2 \end{bmatrix}.$$

$$\det A = \underline{\qquad} \cdot$$

Work.

	ID # :		Name :		
[VI]	(20 pts)	(1) Circle all mat	trices that are in reduced row echelon form	m:	
	(1a) [$\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}.$	(1b) $ \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} . $		
	(1c)	$\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$	(1d) $\begin{bmatrix} 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}.$		
	(1e)	$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}.$	$(1f) \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$		

(2) Make the matrix below a reduced row echelon form, by filling in either 0 or '*' (where * denotes an arbitrary number) to the empty boxes, where all the leading 1s are already shown in the matrix.

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		1			
			1		

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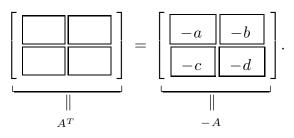
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[VII] (Extra Credit; 10pts) Let
$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$
.

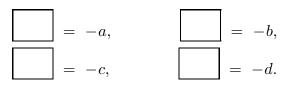
Assume $A^T = -A$. Prove that there is a scalar s such that

$$A = s \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}.$$

Proof. $A^T = -A$ reads



Hence



From these, conclude

$$a =$$
 $d =$ $b =$

Hence

