**Math 105 TOPICS IN MATHEMATICS**

**REVIEW OF LECTURES – IV (SUPPLEMENT)**

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APPENDIX TO §4. GET USED TO LETTERS. SUBSTITUTIONS. SHIFTS.

- **FAQ.** What is the role of letters in mathematics?
- **Answer.** Letters represent numbers.

In other words, a letter is to be substituted by a number.

Today, I only use the letter $n$.

**Example 1.** Substitute $\boxed{n = 5}$ in $\boxed{n + 1}$.

**Solution.** $5 + 1 = 6.$

**Example 2.** Substitute $\boxed{n = 3}$ in $\boxed{n + 2}$.

**Solution.** $3 + 2 = 5.$

**Example 3.** Substitute $\boxed{n = 11}$ in $\boxed{n + 3}$.

**Solution.** $11 + 3 = 14.$

**Example 4.** Substitute $\boxed{n = 20}$ in $\boxed{n + 4}$.

**Solution.** $$20 + 4 = 24.$$

**Example 5.** Substitute $\boxed{n = 1}$ in $\boxed{\frac{1}{2}n\Big(n+1\Big)}$.

**Solution.** $$\frac{1}{2} \cdot 1 \cdot 2 = 1.$$

**Example 6.** Substitute $\boxed{n = 4}$ in $\boxed{\frac{1}{2}n\Big(n+1\Big)}$.

**Solution.** $$\frac{1}{2} \cdot 4 \cdot 5 = 10.$$

**Example 7.** Substitute $\boxed{n = 6}$ in $\boxed{\frac{1}{6}n\Big(n+1\Big)\Big(n+2\Big)}$.

**Solution.** $$\frac{1}{6} \cdot 6 \cdot 7 \cdot 8 = 56.$$

**Example 8.** Substitute $\boxed{n = 9}$ in $\boxed{\frac{1}{24}n\Big(n+1\Big)\Big(n+2\Big)\Big(n+3\Big)}$.

**Solution.** $$\frac{1}{24} \cdot 9 \cdot 10 \cdot 11 \cdot 12 = 495.$$

$\star$ Get used to expressions like

(a) $$1 + 2 + 3 + 4 + 5 + \cdots + n,$$

(b) $$1 + 3 + 6 + 10 + 15 + \cdots + \frac{1}{2}n\Big(n+1\Big),$$

(c) $$1 + 4 + 10 + 15 + 21 + \cdots + \frac{1}{6}n\Big(n+1\Big)\Big(n+2\Big).$$

$\star$ The meaning of (a) is self-evident.

$\star$ (b) is as follows:

$$\underset{n=1}{\underset{\uparrow}{1}} + \underset{n=2}{\underset{\uparrow}{3}} + \underset{n=3}{\underset{\uparrow}{6}} + \underset{n=4}{\underset{\uparrow}{10}} + \underset{n=5}{\underset{\uparrow}{15}} + \cdots + \underset{n=n}{\underset{\uparrow}{\underbrace{\frac{1}{2}n\Big(n+1\Big)}}}.$$

Arrows point the outcomes of substituting the indicated numbers in $\frac{1}{2}n\Big(n+1\Big)$.

$\star$ Similarly, (c) is as follows:

$$\underset{n=1}{\underset{\uparrow}{1}} + \underset{n=2}{\underset{\uparrow}{4}} + \underset{n=3}{\underset{\uparrow}{10}} + \underset{n=4}{\underset{\uparrow}{20}} + \underset{n=5}{\underset{\uparrow}{35}} + \cdots + \underset{n=n}{\underset{\uparrow}{\underbrace{\frac{1}{6}n\Big(n+1\Big)\Big(n+2\Big)}}}.$$

Once again, arrows point the outcomes of substituting the indicated numbers in $\frac{1}{6}n\Big(n+1\Big)\Big(n+2\Big)$.

**Example 9.** Substitute $\boxed{n = 8}$ in $\boxed{1+2+3+4+5+\ \cdots\ +n}$ .

First spell out the outcome, and then calculate it.

**Solution.** $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 \quad = \quad 36.$

**Example 10.** Substitute $\boxed{n = 6}$ in $\boxed{1+3+6+10+\cdots+\frac{1}{2}n\Big(n+1\Big)}$ .

First spell out the outcome, and then calculate it.

**Solution.** $1 + 3 + 6 + 10 + 15 + 21 \quad = \quad 56.$

- **Shift.**

Among important operations in mathematics is the 'shift' of the letter $n$. Shift means you

$$\text{substitute}\ \boxed{n}\ \ \text{by}\ \boxed{n+1}\ .$$

So, every single $n$ spotted in the given formation is getting replaced with $n + 1$.

We denote the shift by $n \mapsto n+1$.

**Example 11.** Shift $\boxed{n \mapsto n+1}$ in $\boxed{n + 3}$ .

**Solution.** $\Big(n+1\Big) + 3 \quad = \quad n + 4.$

**Example 12.** $\underline{\underline{\text{Shift}}}$ $\boxed{n \mapsto n+1}$ $\underline{\underline{\text{in}}}$ $\boxed{n + 10}$ .

**Solution.**

$$\Big(n+1\Big) + 10 \quad = \quad n + 11.$$

**Example 13.** $\underline{\underline{\text{Shift}}}$ $\boxed{n \mapsto n+1}$ $\underline{\underline{\text{in}}}$ $\boxed{\frac{1}{2} n\Big(n+1\Big)}$ .

**Solution.**

$$\frac{1}{2}\Big(n+1\Big)\Big(n+2\Big).$$

**Example 14.** $\underline{\underline{\text{Shift}}}$ $\boxed{n \mapsto n+1}$ $\underline{\underline{\text{in}}}$ $\boxed{\frac{1}{6} n\Big(n+1\Big)\Big(n+2\Big)}$ .

**Solution.**

$$\frac{1}{6}\Big(n+1\Big)\Big(n+2\Big)\Big(n+3\Big).$$

**Example 15.** $\underline{\underline{\text{Shift}}}$ $\boxed{n \mapsto n+1}$ $\underline{\underline{\text{in}}}$

$$\boxed{1 + 3 + 6 + 10 + 15 + \cdots + \frac{1}{2} n\Big(n+1\Big)} \; .$$

**Solution.**

$$1 + 3 + 6 + 10 + 15 + \cdots + \frac{1}{2}\Big(n+1\Big)\Big(n+2\Big).$$