Math 105 TOPICS IN MATHEMATICS
REVIEW OF LECTURES – XXXI

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§31. Trigonometry – II.

Today we only cover one thing — special values of sin, and cos. Last time I said making precise the notion of angles is a taxing job. That will be the subject of the next lecture. Today, we *pretend* that we already know about angles. Today it suffices to know the following 'provisional' definition, which will be ultimately replaced:

" *Angle is an enumerable quantity that apportions the extent of the margin subtended by two rays that share a common end point* ".

Accordingly, we rely on our *empirical* knowledge what two adjacent angles mean, the flat angle is $180°$, the right angle is $90°$, what bisecting and trisecting angles mean, *etc.* Today's material only involves the degrees

$$0°, \qquad 30°, \qquad 45°, \qquad 60°, \qquad 90°,$$

and $90°$ added to or subtracted from them. You stick with those angles and the rigor issue is not quite going to be at stake. So, count that the next lecture is more challenging whereas today's lecture is on the elementary side. So, I repeat: Throughout today's lecture, what I refer to as an angle is something that is based on our geometric intuition. We never think twice about its rigor.

As for 'degrees versus radians': We first work with the 'degrees' and then we translate everything into 'radians'. So today we touch both. Like we did last time, let's work with the xy-coordiante system. O is the coordinate origin. P be a point lying in the unit circle. So, $\angle POP_0$ is θ. Suppose P resides in the upper-half of the coordinate system, then $0° \le \theta \le 180°$. On the other hand, suppose P resides in the lower-half of the coordinate system, then $-180° \le \theta \le 0°$. Now, the x-coordinate reading of the point P is $\cos\theta$, the y-coordinate reading of the point P is $\sin\theta$.

$$x = \cos\theta, \qquad\qquad y = \sin\theta.$$

You are free to add or subtract $360°$ to an angle and neither its sin-value nor its cos-value changes.

You need to by all means understand and memorize the following (the separate picture file helps):

- **Basic 'cos' and 'sin' values – I.** $\mathbf{0° \leq \theta \leq 90°}$.

$$\cos 0° = 1, \qquad \sin 0° = 0,$$

$$\cos 30° = \frac{\sqrt{3}}{2}, \qquad \sin 30° = \frac{1}{2},$$

$$\cos 45° = \frac{1}{\sqrt{2}} \left(= \frac{\sqrt{2}}{2}\right), \qquad \sin 45° = \frac{1}{\sqrt{2}} \left(= \frac{\sqrt{2}}{2}\right),$$

$$\cos 60° = \frac{1}{2}, \qquad \sin 60° = \frac{\sqrt{3}}{2},$$

$$\cos 90° = 0, \qquad \sin 90° = 1.$$

- **Basic 'cos' and 'sin' values – II.** $\mathbf{90° < \theta \leq 180°}$.

$$\cos 120° = -\frac{1}{2}, \qquad \sin 120° = \frac{\sqrt{3}}{2},$$

$$\cos 135° = -\frac{1}{\sqrt{2}} \left(= -\frac{\sqrt{2}}{2}\right), \qquad \sin 135° = \frac{1}{\sqrt{2}} \left(= \frac{\sqrt{2}}{2}\right),$$

$$\cos 150° = -\frac{\sqrt{3}}{2}, \qquad \sin 150° = \frac{1}{2},$$

$$\cos 180° = -1, \qquad \sin 180° = 0.$$

- **Basic 'cos' and 'sin' values – III. $-90° \leq \theta < 0°$.**

$$\cos\left(-30°\right) = \frac{\sqrt{3}}{2}, \qquad \sin\left(-30°\right) = -\frac{1}{2},$$

$$\cos\left(-45°\right) = \frac{1}{\sqrt{2}} \left(= \frac{\sqrt{2}}{2}\right), \qquad \sin\left(-45°\right) = -\frac{1}{\sqrt{2}} \left(= -\frac{\sqrt{2}}{2}\right),$$

$$\cos\left(-60°\right) = \frac{1}{2}, \qquad \sin\left(-60°\right) = -\frac{\sqrt{3}}{2},$$

$$\cos\left(-90°\right) = 0, \qquad \sin\left(-90°\right) = -1.$$

- **Basic 'cos' and 'sin' values – IV. $-180° < \theta < -90°$.**

$$\cos\left(-120°\right) = -\frac{1}{2}, \qquad \sin\left(-120°\right) = -\frac{\sqrt{3}}{2},$$

$$\cos\left(-135°\right) = -\frac{1}{\sqrt{2}} \left(= -\frac{\sqrt{2}}{2}\right), \qquad \sin\left(-135°\right) = -\frac{1}{\sqrt{2}} \left(= -\frac{\sqrt{2}}{2}\right),$$

$$\cos\left(-150°\right) = -\frac{\sqrt{3}}{2}, \qquad \sin\left(-150°\right) = -\frac{1}{2},$$

$$\cos\left(-180°\right) = -1, \qquad \sin\left(-180°\right) = 0.$$

- Now, check out the conversion table in the next page:

radian	degree
0	0°
$\frac{\pi}{6}$	30°
$\frac{\pi}{4}$	45°
$\frac{\pi}{3}$	60°
$\frac{\pi}{2}$	90°
$\frac{2\pi}{3}$	120°
$\frac{3\pi}{4}$	135°
$\frac{5\pi}{6}$	150°
π	180°
$-\frac{\pi}{6}$	-30°
$-\frac{\pi}{4}$	-45°
$-\frac{\pi}{3}$	-60°
$-\frac{\pi}{2}$	-90°
$-\frac{2\pi}{3}$	-120°
$-\frac{3\pi}{4}$	-135°
$-\frac{5\pi}{6}$	-150°
$-\pi$	-180°

This allows us to rewritre everything using radians:

- **Basic 'cos' and 'sin' values (in radians) – I.** $\boldsymbol{0 \leq \theta \leq \frac{\pi}{2}}$**.**

$$\cos\ 0 = 1, \qquad \sin\ 0 = 0,$$

$$\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}, \qquad \sin\frac{\pi}{6} = \frac{1}{2},$$

$$\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}} \left(= \frac{\sqrt{2}}{2}\right), \qquad \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}} \left(= \frac{\sqrt{2}}{2}\right),$$

$$\cos\frac{\pi}{3} = \frac{1}{2}, \qquad \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2},$$

$$\cos\frac{\pi}{2} = 0, \qquad \sin\frac{\pi}{2} = 1.$$

- **Basic 'cos' and 'sin' values (in radians) – II.** $\boldsymbol{\frac{\pi}{2} < \theta \leq \pi}$**.**

$$\cos\frac{2\pi}{3} = -\frac{1}{2}, \qquad \sin\frac{2\pi}{3} = \frac{\sqrt{3}}{2},$$

$$\cos\frac{3\pi}{4} = -\frac{1}{\sqrt{2}} \left(= -\frac{\sqrt{2}}{2}\right), \qquad \sin\frac{3\pi}{4} = \frac{1}{\sqrt{2}} \left(= \frac{\sqrt{2}}{2}\right),$$

$$\cos\frac{5\pi}{6} = -\frac{\sqrt{3}}{2}, \qquad \sin\frac{5\pi}{6} = \frac{1}{2},$$

$$\cos\ \pi = -1. \qquad \sin\ \pi = 0.$$

- **Basic 'cos' and 'sin' values (in radians) – III.** $\boldsymbol{-\frac{\pi}{2} \leq \theta < 0.}$

$$\cos\left(-\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}, \qquad \sin\left(-\frac{\pi}{6}\right) = -\frac{1}{2},$$

$$\cos\left(-\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}\left(= \frac{\sqrt{2}}{2}\right), \qquad \sin\left(-\frac{\pi}{4}\right) = -\frac{1}{\sqrt{2}}\left(= -\frac{\sqrt{2}}{2}\right),$$

$$\cos\left(-\frac{\pi}{3}\right) = \frac{1}{2}, \qquad \sin\left(-\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2},$$

$$\cos\left(-\frac{\pi}{2}\right) = 0, \qquad \sin\left(-\frac{\pi}{2}\right) = -1.$$

- **Basic 'cos' and 'sin' values (in radians) – IV.** $\boldsymbol{-\pi \leq \theta < \frac{\pi}{2}.}$

$$\cos\left(-\frac{2\pi}{3}\right) = -\frac{1}{2}, \qquad \sin\left(-\frac{2\pi}{3}\right) = -\frac{\sqrt{3}}{2},$$

$$\cos\left(-\frac{3\pi}{4}\right) = \frac{1}{\sqrt{2}}\left(= \frac{\sqrt{2}}{2}\right), \qquad \sin\left(-\frac{3\pi}{4}\right) = -\frac{1}{\sqrt{2}}\left(= -\frac{\sqrt{2}}{2}\right),$$

$$\cos\left(-\frac{5\pi}{6}\right) = -\frac{\sqrt{3}}{2}, \qquad \sin\left(-\frac{5\pi}{6}\right) = -\frac{1}{2},$$

$$\cos\left(-\pi\right) = -1, \qquad \sin\left(-\pi\right) = 0.$$