

**Math 105 TOPICS IN MATHEMATICS**  
**REVIEW OF LECTURES – XV (SUPPLEMENT)**

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APPENDIX TO §15. ADDING FRACTIONS.

1. How much is

$$\frac{1}{2} + \frac{1}{3} = ?$$

If you add up the two numerators, as in  $1 + 1 = 2$ , and then add up the two denominators, as in  $2 + 3 = 5$ , then you are on the wrong track. Indeed, if you say  $\frac{2}{5}$  is the answer, then I say that's a wrong answer. Below is the right way to do it:

[**Solution**]: Below is called 'common denominator technique':

**Step 1.** Find a common multiple of 2 and 3. It is  $2 \cdot 3 = 6$ .

**Step 2.** Multiply a suitable common integer to the numerator and the denominator of the fraction  $\frac{1}{2}$  so as to make the denominator 6.

$$\begin{aligned}\frac{1}{2} &= \frac{1 \cdot 3}{2 \cdot 3} \\ &= \frac{3}{6}.\end{aligned}$$

**Step 3.** Multiply a suitable common integer to the numerator and the denominator of the fraction  $\frac{1}{3}$  so as to make the denominator 6.

$$\begin{aligned}\frac{1}{3} &= \frac{1 \cdot 2}{3 \cdot 2} \\ &= \frac{2}{6}.\end{aligned}$$

**Step 4.** Now you can add up the outcome of Step 2 and the outcome of step 3:

$$\frac{3}{6} + \frac{2}{6}.$$

This is just

$$\begin{aligned}\frac{3}{6} + \frac{2}{6} &= \frac{3 + 2}{6} \quad (\text{Common denominator}) \\ &= \frac{5}{6}.\end{aligned}$$

So, the answer is  $\frac{5}{6}$ . To highlight:

$$\frac{1}{2} + \frac{1}{3} = \frac{5}{6}.$$

★ A recommended (veteran) way of writing the answer:

$$\begin{aligned}\frac{1}{2} + \frac{1}{3} &= \frac{1 \cdot 3}{2 \cdot 3} + \frac{1 \cdot 2}{3 \cdot 2} \\ &= \frac{3}{6} + \frac{2}{6} \\ &= \frac{3 + 2}{6} \\ &= \frac{5}{6}.\end{aligned}$$

2. How much is

$$\frac{7}{20} - \frac{4}{15} = ?$$

Once again, you don't want to do it like  $7 - 4$  and  $20 - 15$ . That's a wrong track. Right track is, as always, 'common denominator' technique.

[Solution]: Below is called 'common denominator technique':

**Step 1.** Find a common multiple of 20 and 15. It is  $20 \cdot 3 = 15 \cdot 4 = 60$ .

**Step 2.** Multiply a suitable common integer to the numerator and the denominator of the fraction  $\frac{7}{20}$  so as to make the denominator 60.

$$\begin{aligned}\frac{7}{20} &= \frac{7 \cdot 3}{20 \cdot 3} \\ &= \frac{21}{60}.\end{aligned}$$

**Step 3.** Multiply a suitable common integer to the numerator and the denominator of the fraction  $\frac{4}{15}$  so as to make the denominator 60.

$$\begin{aligned}\frac{4}{15} &= \frac{4 \cdot 4}{15 \cdot 4} \\ &= \frac{16}{60}.\end{aligned}$$

**Step 4.** Now you can subtract the outcome of Step 3 from the outcome of step 2:

$$\frac{21}{60} - \frac{16}{60}.$$

This is just

$$\begin{aligned}\frac{21}{60} - \frac{16}{60} &= \frac{21 - 16}{60} && \text{(Common denominator)} \\ &= \frac{5}{60}.\end{aligned}$$

**Step 5.** Simplify:

$$\begin{aligned}\frac{5}{60} &= \frac{1 \cdot 5}{12 \cdot 5} \\ &= \frac{1}{12}.\end{aligned}$$

So, the answer is  $\frac{1}{12}$ . To highlight:

$$\frac{7}{20} - \frac{4}{15} = \frac{1}{12}.$$

★ A recommended (veteran) way of writing the answer:

$$\begin{aligned}\frac{7}{20} - \frac{4}{15} &= \frac{7 \cdot 3}{20 \cdot 3} - \frac{4 \cdot 4}{15 \cdot 4} \\ &= \frac{21}{60} - \frac{16}{60} \\ &= \frac{21 - 16}{60} \\ &= \frac{5}{60} \\ &= \frac{1 \cdot 5}{12 \cdot 5} \\ &= \frac{1}{12}.\end{aligned}$$

**Exercise 1.** Find

$$(a) \quad \frac{3}{2} + \frac{7}{4}. \quad (b) \quad \frac{2}{7} + \frac{9}{10}. \quad (c) \quad \frac{5}{12} + \frac{11}{30}.$$

$$(a)' \quad \frac{3}{2} - \frac{7}{4}. \quad (b)' \quad \frac{2}{7} - \frac{9}{10}. \quad (c)' \quad \frac{5}{12} - \frac{11}{30}.$$

$$\left[ \underline{\text{Answers}} \right]: \quad (a) \quad \frac{13}{4}. \quad (b) \quad \frac{83}{70}. \quad (c) \quad \frac{47}{60}.$$

$$(a)' \quad -\frac{1}{4}. \quad (b)' \quad -\frac{43}{70}. \quad (c)' \quad \frac{1}{20}.$$

• Calculate

$$(1) \quad 1 = ?$$

$$(2) \quad 1 + \frac{1}{2} = ?$$

$$(3) \quad 1 + \frac{1}{2} + \frac{1}{3} = ?$$

$$(4) \quad 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} = ?$$

$$(5) \quad 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} = ?$$

$$(6) \quad 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} = ?$$

$\left[ \underline{\text{Solution}} \right]:$  First, the answer for (1) is clearly 1.

As for (2),

$$\begin{aligned}1 + \frac{1}{2} &= \frac{2}{2} + \frac{1}{2} \\ &= \frac{1 + 2}{2} \\ &= \frac{3}{2}.\end{aligned}$$

As for (3), you add up  $\frac{3}{2}$ , which is the answer for (2), and  $\frac{1}{3}$ :

$$\begin{aligned}\frac{3}{2} + \frac{1}{3} &= \frac{3 \cdot 3}{2 \cdot 3} + \frac{1 \cdot 2}{3 \cdot 2} \\ &= \frac{9}{6} + \frac{2}{6} \\ &= \frac{9 + 2}{6} \\ &= \frac{11}{6}.\end{aligned}$$

As for (4), you add up  $\frac{11}{6}$ , which is the answer for (3), and  $\frac{1}{4}$ :

$$\begin{aligned}\frac{11}{6} + \frac{1}{4} &= \frac{11 \cdot 2}{6 \cdot 2} + \frac{1 \cdot 3}{4 \cdot 3} \\ &= \frac{22}{12} + \frac{3}{12} \\ &= \frac{22 + 3}{12} \\ &= \frac{25}{12}.\end{aligned}$$

As for (5), you add up  $\frac{25}{12}$ , which is the answer for (4), and  $\frac{1}{5}$ :

$$\begin{aligned}\frac{25}{12} + \frac{1}{5} &= \frac{25 \cdot 5}{12 \cdot 5} + \frac{1 \cdot 12}{5 \cdot 12} \\ &= \frac{125}{60} + \frac{12}{60} \\ &= \frac{125 + 12}{60} \\ &= \frac{137}{60}.\end{aligned}$$

As for (6), you add up  $\frac{137}{60}$ , which is the answer for (5), and  $\frac{1}{6}$ :

$$\begin{aligned}\frac{137}{60} + \frac{1}{6} &= \frac{137}{60} + \frac{1 \cdot 10}{6 \cdot 10} \\ &= \frac{137}{60} + \frac{10}{60} \\ &= \frac{137 + 10}{60} \\ &= \frac{147}{60} \\ &= \frac{49 \cdot 3}{20 \cdot 3} \\ &= \frac{49}{20}.\end{aligned}$$

★ To summarize the result:

$$(1) \quad 1 = 1,$$

$$(2) \quad 1 + \frac{1}{2} = \frac{3}{2},$$

$$(3) \quad 1 + \frac{1}{2} + \frac{1}{3} = \frac{11}{6},$$

$$(4) \quad 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \frac{25}{12},$$

$$(5) \quad 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} = \frac{137}{60},$$

$$(6) \quad 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} = \frac{49}{20}.$$

**Exercise 2.**

$$(7) \quad 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} = ?$$

$$(8) \quad 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} = ?$$

$$(9) \quad 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} + \frac{1}{9} = ?$$

$$(10) \quad 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} + \frac{1}{9} + \frac{1}{10} = ?$$

$$\left[ \underline{\text{Answers}} \right]: \quad (7) \quad \frac{363}{140}. \quad (8) \quad \frac{761}{280}. \quad (9) \quad \frac{7129}{2520}.$$

$$(10) \quad \frac{7381}{2520}.$$