Math 105 TOPICS IN MATHEMATICS
SOLUTION FOR QUIZ – VIII (03/27)

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   Line #:   52920.

[I] (4pts)   Convert each of the following expression of numbers in the binary system back into the usual decimal system.

(a)   11.   (b)   110.   (c)   1001.   (d)   1111111 (seven straight 1s).

$\boxed{\textbf{Answers}}$:   (a)   3.   (b)   6.   (c)   9.   (d)   127 $\left(= 2^7 - 1\right)$.

[II] (3pts)   Convert each of the following expressions of numbers in the usual decimal system into the binary system.

(a)   5.   (b)   12.   (c)   35.

$\boxed{\textbf{Answers}}$:   (a)   101.   (b)   1100.   (c)   100011.

[III] (3pts)   Convert each of the following expression of numbers in the hexadecimal system back into the usual decimal system.

(a) C.   (b) $1B$.   (c) $A5$.

$\boxed{\textbf{Answers}}$:   (a)   12.   (b)   27.   (c)   165.

[IV] (3pts)   Convert each of the following expressions of numbers in the usual decimal system into the hexadecimal system.

(a)   16.   (b)   28.   (c) $16^4 - 1$.

$\boxed{\textbf{Answers}}$:   (a)   10.   (b) $1C$.   (c) $FFFF$.

[V] (3pts)   Simplify:

(1) $3^x \cdot 8^x = 24^x.$ (2) $a^{\frac{1}{3}} \cdot a^{\frac{5}{3}} = a^2.$

(3) $\left(a^{\sqrt{3}}\right)^{\sqrt{12}} = a^6.$

[VI] (2pts)   Find the limits:

(1) $\displaystyle\lim_{n \to \infty} \left(1 + \frac{8}{n}\right)^n = e^8.$ (2) $\displaystyle\lim_{n \to \infty} \left(1 - \frac{1}{2n}\right)^n = e^{-\frac{1}{2}}.$

[VII] (3pts)   Identify the following infinite sum $\left(\text{the answer is an } e\text{-to-the-power}\right)$:

$$1 + \frac{1}{1!} \cdot 3 + \frac{1}{2!} \cdot 3^2 + \frac{1}{3!} \cdot 3^3 + \frac{1}{4!} \cdot 3^4 + \frac{1}{5!} \cdot 3^5 + \cdots = e^3.$$