Math 105 TOPICS IN MATHEMATICS

SOLUTION FOR QUIZ – VI (03/02)

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[I] (3pts) $\dfrac{1}{2}+\dfrac{1}{4}+\dfrac{1}{8}+\dfrac{1}{16}+\dfrac{1}{32}+\dfrac{1}{64}+\dfrac{1}{128} = \dfrac{127}{128}$.

$\left(\text{Also,}\quad 1-\dfrac{1}{128}\quad \text{is acceptable as an answer.}\right)$

[II] (3pts) Which one is bigger?

(a) $\left(1+\dfrac{1}{20}\right)^{20}$ or

(b) $1+\dfrac{1}{1!}+\dfrac{1}{2!}+\dfrac{1}{3!}+\dfrac{1}{4!}+\dfrac{1}{5!}+\dfrac{1}{6!}+\dfrac{1}{7!}+\dfrac{1}{8!}+\dfrac{1}{9!}+\dfrac{1}{10!}$
$+\dfrac{1}{11!}+\dfrac{1}{12!}+\dfrac{1}{13!}+\dfrac{1}{14!}+\dfrac{1}{15!}+\dfrac{1}{16!}+\dfrac{1}{17!}+\dfrac{1}{18!}+\dfrac{1}{19!}+\dfrac{1}{20!}$.

$\left[\underline{\textbf{Answer}}\right]$**:** (b) is bigger. $\left(\text{This is clear once you binomially expand (a).}\right)$

[III] (3pts) $e = \lim_{n\to\infty}\left(1+\boxed{\dfrac{1}{n}}\right)^{n}$.

[IV] (6pts) (a) $\dfrac{1}{945}$ is $\underline{\text{a rational number}}$.

(b) 14.77777777777777777... $\left(\text{the digit } 7 \text{ continues permanently}\right)$ is $\underline{\text{a rational number}}$. $\left(\text{This is } 14+\dfrac{7}{9}=\dfrac{133}{9}.\right)$

[V] (5pts) Do $\sqrt{2}$ and

$$1+\frac{24}{60}+\frac{51}{60^2}+\frac{10}{60^3}+\frac{7}{60^4}+\frac{46}{60^5}+\frac{6}{60^6}+\frac{4}{60^7}+\frac{44}{60^8}+\frac{50}{60^9}+\frac{28}{60^{10}}+\frac{51}{60^{11}}+\frac{20}{60^{12}}$$

coincide as real numbers?

[Answer]: No, they do not coincide.

Explain. $\sqrt{2}$ is an irrational number, whereas the following is a rational number:

$$1+\frac{24}{60}+\frac{51}{60^2}+\frac{10}{60^3}+\frac{7}{60^4}+\frac{46}{60^5}+\frac{6}{60^6}+\frac{4}{60^7}+\frac{44}{60^8}+\frac{50}{60^9}+\frac{28}{60^{10}}+\frac{51}{60^{11}}+\frac{20}{60^{12}}.$$

⋆ **[Note]** :

$$\sqrt{2} = 1.41421356237309504880168872420969 81...\ ,$$

$$1+\frac{24}{60}+\frac{51}{60^2}+\frac{10}{60^3}+\frac{7}{60^4}+\frac{46}{60^5}+\frac{6}{60^6}+\frac{4}{60^7}+\frac{44}{60^8}+\frac{50}{60^9}+\frac{28}{60^{10}}+\frac{51}{60^{11}}+\frac{20}{60^{12}}$$

$$= 1.41421356237309504880142504059716 88...\ .$$