Math 105 TOPICS IN MATHEMATICS

SOLUTION FOR QUIZ – III (02/13)

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[I] (6pts) (1) Which number is bigger, $\frac{1}{15}$, or $\frac{1}{150}$?

[Answer]: $\frac{1}{15}$.

(2) Which number is bigger, $\frac{23}{24}$, or $\frac{24}{25}$?

[Answer]: $\frac{24}{25}$.

(3) Which number is bigger, $\frac{98}{99} \cdot \frac{97}{99}$, or $\frac{99}{100} \cdot \frac{98}{100}$?

[Answer]: $\frac{99}{100} \cdot \frac{98}{100}$.

[II] (6pts) (1) True or false :

If $\boxed{a < b}$ and moreover a is positive, then $\boxed{\frac{1}{a} < \frac{1}{b}}$.

[Answer]: False.

(2) True or false : Suppose all of a, b, c, d are positive.

If $\boxed{a < b}$ and $\boxed{c < d}$ then $\boxed{a\,c < b\,d}$.

[Answer]: True.

[III] (3pts)

$$\binom{9}{4}\cdot\left(\frac{1}{9}\right)^4 \;=\; \frac{1}{\boxed{1}\cdot\boxed{2}\cdot\boxed{3}\cdot\boxed{4}}\cdot\frac{\boxed{9}}{\boxed{9}}\cdot\frac{\boxed{8}}{\boxed{9}}\cdot\frac{\boxed{7}}{\boxed{9}}\cdot\frac{\boxed{6}}{\boxed{9}}.$$

$$\binom{40}{3}\cdot\left(\frac{1}{40}\right)^3 \;=\; \frac{1}{\boxed{1}\cdot\boxed{2}\cdot\boxed{3}}\cdot\frac{\boxed{40}}{\boxed{40}}\cdot\frac{\boxed{39}}{\boxed{40}}\cdot\frac{\boxed{38}}{\boxed{40}}.$$

[IV] (5pts) Which number among the following (a), (b) and (c) is the biggest?

(a) $\left(1+\frac{1}{10^5}\right)^{10^5}$, (b) $\left(1+\frac{1}{10^{10}}\right)^{10^{10}}$, or (c) $\left(1+\frac{1}{10^{15}}\right)^{10^{15}}$.

$\left[\underline{\text{Answer}}\right]$: (c) $\left(1+\frac{1}{10^{15}}\right)^{10^{15}}$.