

Math 105 TOPICS IN MATHEMATICS

SOLUTION FOR QUIZ – X (04/06)

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[1] (6pts) (1) $(x^2 - 4)^2 = x^4 - 8x^2 + 16.$

(2) $(x^2 + 4x + 2)^2 = x^4 + 8x^3 + 20x^2 + 16x + 4.$

Work.

$$\begin{array}{r} \phantom{} \\ \phantom{} \\ \hline \phantom{} \\ \\ \\ \hline \end{array}$$

(3) $(1 + x + x^2 + x^3 + x^4)^2$
 $= 1 + 2x + 3x^2 + 4x^3 + 5x^4 + 4x^5 + 3x^6 + 2x^7 + x^8.$

Work.

$$\begin{array}{r} \phantom{} \\ \phantom{} \\ \hline \phantom{} \\ \phantom{} \\ \phantom{} \\ \phantom{} \\ \phantom{} \\ \phantom{} \\ \phantom{} \\ \phantom{} \\ \phantom{} \\ \hline \end{array}$$

$$\begin{aligned} \text{[II] (2pts)} \quad (x-2)(x+3)^2 &= (x-2)(x^2+6x+9) \\ &= x^3 + 4x^2 - 3x - 18. \end{aligned}$$

Work.

$$\begin{array}{r} \phantom{\underline{\hspace{1.5cm}}} \phantom{\hspace{2cm}} x^2 + 6x + 9 \\ \phantom{\underline{\hspace{1.5cm}}} \phantom{\hspace{2cm}} + 6x + 9 \\ \times) \phantom{\underline{\hspace{1.5cm}}} \phantom{\hspace{2cm}} + 6x + 9 \\ \hline \phantom{\underline{\hspace{1.5cm}}} \phantom{\hspace{2cm}} - 2x^2 - 12x - 18 \\ \phantom{\underline{\hspace{1.5cm}}} \phantom{\hspace{2cm}} x^3 + 6x^2 + 9x \\ \hline \phantom{\underline{\hspace{1.5cm}}} \phantom{\hspace{2cm}} x^3 + 4x^2 - 3x - 18 \end{array}$$

$$\begin{aligned} \text{[III] (2pts)} \quad (x-1)(x+1)(x^2+1)(x^4+1) &= (x^2-1)(x^2+1)(x^4+1) \\ &= (x^4-1)(x^4+1) \\ &= x^8-1. \end{aligned}$$

[IV] (6pts)

$$\begin{aligned} (1) \quad (x^2 - \sqrt{2}x + 1)(x^2 + \sqrt{2}x + 1)(x^4 - 1) &= \left((x^2 + 1) - \sqrt{2}x \right) \left((x^2 + 1) + \sqrt{2}x \right) (x^4 - 1) \\ &= \left((x^2 + 1)^2 - (\sqrt{2}x)^2 \right) (x^4 - 1) \\ &= \left((x^4 + 2x^2 + 1) - 2x^2 \right) (x^4 - 1) \\ &= (x^4 + 1)(x^4 - 1) = x^8 - 1. \end{aligned}$$

$$\begin{aligned}
(2) \quad & (x - \sqrt{2} - \sqrt{3})(x + \sqrt{2} - \sqrt{3})(x - \sqrt{2} + \sqrt{3})(x + \sqrt{2} + \sqrt{3}) \\
&= \left((x - \sqrt{3}) - \sqrt{2} \right) \left((x - \sqrt{3}) + \sqrt{2} \right) \\
&\quad \cdot \left((x + \sqrt{3}) - \sqrt{2} \right) \left((x + \sqrt{3}) + \sqrt{2} \right) \\
&= \left((x - \sqrt{3})^2 - (\sqrt{2})^2 \right) \left((x + \sqrt{3})^2 - (\sqrt{2})^2 \right) \\
&= \left(x^2 - 2\sqrt{3}x + 3 - 2 \right) \left(x^2 + 2\sqrt{3}x + 3 - 2 \right) \\
&= \left(x^2 - 2\sqrt{3}x + 1 \right) \left(x^2 + 2\sqrt{3}x + 1 \right) \\
&= \left((x^2 + 1) - 2\sqrt{3}x \right) \left((x^2 + 1) + 2\sqrt{3}x \right) \\
&= \left(x^2 + 1 \right)^2 - \left(2\sqrt{3}x \right)^2 \\
&= \left(x^4 + 2x^2 + 1 \right) - \left(12x^2 \right) = x^4 - 10x^2 + 1.
\end{aligned}$$

[V] (4pts)

$$\begin{aligned}
& (x + 1)(x + 2)(x + 3)(x + 4)(x + 5)(x + 6)(x + 7) \\
&= x^7 + 28x^6 + 322x^5 + 1960x^4 + 6769x^3 + 13132x^2 + 13068x + 5040.
\end{aligned}$$

Work for [V].

