Math 105 TOPICS IN MATHEMATICS
SOLUTION FOR REGULAR HOMEWORK – IX (04/06)

<div align="right">April 8 (Wed), 2015</div>

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[I] (3pts)   Permute the order of terms, if necessary, to make each of the given polynomials in the <u>ascending</u> order.

(1) $5x + x^3 - 2x^2.$ (2) $\dfrac{1}{3} - 4x^5 + 2x^4 - \dfrac{1}{2}x.$

(3) $x^{10} + x^9 + x^8 + x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1.$

$\left[\,\underline{\textbf{Answers}}\,\right]$:   (1) $5x - 2x^2 + x^3.$ (2) $\dfrac{1}{3} - \dfrac{1}{2}x + 2x^4 - 4x^5.$

(3) $1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + x^8 + x^9 + x^{10}.$

[II] (3pts)   Permute the order of terms, if necessary, to make each of the given polynomials in the <u>descending</u> order.

(1) $2x^2 - \sqrt{2}\,x^4 - \sqrt{2}.$ (2) $7x^5 + 4x^6 + 10x^7 - 13x^8.$

(3) $x^2 - x^4 + x^6 - x^8.$

$\left[\,\underline{\textbf{Answers}}\,\right]$:   (1) $-\sqrt{2}\,x^4 + 2x^2 - \sqrt{2}.$ (2) $-13x^8 + 10x^7 + 4x^6 + 7x^5.$

(3) $-x^8 + x^6 - x^4 + x^2.$

[III] (4pts)   (1) $\left(2x^3 + 3x^2 + 4x\right) + \left(x^4 + 5\right)$ $=$ $x^4 + 2x^3 + 3x^2 + 4x + 5.$

(2) $\left(x^4 + 8x^3 + 4x\right) - \left(x^4 + 7x^3 - 3x^2 + 6\right)$ $=$ $x^3 + 3x^2 + 4x - 6.$

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(3) $f\left(x\right) = x^8 + 7x^5 + 21x^2,$ $g\left(x\right) = x^6 - 3x^4 - 10x^2$

$\Longrightarrow$ $f\left(x\right) + g\left(x\right) = x^8 + x^6 + 7x^5 - 3x^4 + 11x^2.$

(4) $f\left(x\right) = \dfrac{1}{3}x^4 + \dfrac{1}{2}x^2 + 1,$ $g\left(x\right) = \dfrac{2}{3}x^4 + \dfrac{3}{2}x^2 + \dfrac{5}{6}$

$\Longrightarrow$ $f\left(x\right) - g\left(x\right) = -\dfrac{1}{3}x^4 - x^2 + \dfrac{1}{6}.$

[IV] (3pts)   (1) $-10\left(81x^6 - 162x^3 + 729\right) = -810x^6 + 1620x^3 - 7290.$

(2) $2\left(\dfrac{1}{2}x^3 + \dfrac{9}{2}x^2 + \dfrac{17}{2}x\right) = x^3 + 9x^2 + 17x.$

(3) $\dfrac{6x^5 - 36x^4 + 48x^3 - 48x^2 + 36x - 6}{6}$

$= x^5 - 6x^4 + 8x^3 - 8x^2 + 6x - 1.$

[V] (3pts)   (1) $x\left(14x^3 + 21x^2 + 35x\right) = 14x^4 + 21x^3 + 35x^2.$

(2) $\dfrac{3}{4}x\left(6x^5 + 10x^4 + 12x^3 + 8x^2 + 4x\right) = \dfrac{9}{2}x^6 + \dfrac{15}{2}x^5 + 9x^4 + 6x^3 + 3x^2.$

(3) $x^{10}\left(x^6 + x^4 + x^2 + 1\right) = x^{16} + x^{14} + x^{12} + x^{10}.$

[VI] (2pts)   (1) $\left(x + 2\right)\left(x + 9\right) = x^2 + 11x + 18.$

(2) $\left(x + 7\right)\left(x - 4\right) = x^2 + 3x - 28.$

[VII] (4pts)   (1) $\left(x^2 + 3\right)\left(2x^2 + 4x + 1\right) = 2x^4 + 4x^3 + 7x^2 + 12x + 3.$

(2) $\left(x^3 + x^2 + 3x\right)\left(x^2 + 4x - 1\right) = x^5 + 5x^4 + 6x^3 + 11x^2 - 3x.$

[VIII] (8pts)

(1) $\left(x - 1\right)\big(x^{18} + x^{17} + x^{16} + x^{15} + x^{14} + x^{13} + x^{12} + x^{11} + x^{10}$

$\qquad + x^9 + x^8 + x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1\big)$

$= \quad x^{19} - 1.$

(2) $\left(1 - x\right)\big(1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + x^8 + x^9 + x^{10}$

$\qquad + x^{11} + x^{12} + x^{13} + x^{14} \big)$

$= \quad 1 - x^{15}.$

(3) $\left(1 + x\right)\big(1 - x + x^2 - x^3 + x^4 - x^5 + x^6 - x^7 + x^8 - x^9 + x^{10}$

$\qquad - x^{11} + x^{12} - x^{13} + x^{14} - x^{15} + x^{16} \big)$

$= \quad 1 + x^{17}.$

(4) $\left(1 + x\right)\big(1 - x + x^2 - x^3 + x^4 - x^5 + x^6 - x^7 + x^8 - x^9 \big)$

$= \quad 1 - x^{10}.$