

Math 105 TOPICS IN MATHEMATICS
SOLUTION FOR REGULAR HOMEWORK – VIII (03/25)

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[I] (4pts) Convert each of the following expression of numbers in the binary system back into the usual decimal system.

(1) 101. (2) 10011. (3) 100110. (4) 11111111 (nine straight 1s).

[Answers]: (1) 5. (2) 19. (3) 38. (4) $2^9 - 1$ ($= 511$).

[II] (4pts) Convert each of the following expressions of numbers in the usual decimal system into the binary system.

(1) 6. (2) 10. (3) 33. (4) 63.

[Answers]: (1) 110. (2) 1010. (3) 100001. (4) 111111.

[III] (4pts) Convert each of the following expression of numbers in the hexadecimal system back into the usual decimal system.

(1) *D*. (2) *2B*. (3) *AA*. (4) *FF*.

[Answers]: (1) 13. (2) 43. (3) 170. (4) 255.

[IV] (4pts) Convert each of the following expressions of numbers in the usual decimal system into the hexadecimal system.

(1) 10. (2) 15. (3) 29. (4) 4095.

[**Answers**]: (1) *A*. (2) *F*. (3) *1D*. (4) *FFF*.

[V] (5pts) (1) $3^x \cdot 7^x = 21^x$. (2) $a^4 \cdot a^{10} = a^{14}$.

(3) $(a^{\sqrt{3}})^{\sqrt{3}} = a^3$. (4) $1^e = 1$.

(5) $a^{-\sqrt{11}} = \frac{1}{\boxed{a^{\sqrt{11}}}}$.

[VI] (6pts) Find the limits:

(1) $\lim_{n \rightarrow \infty} \left(1 + \frac{5}{n}\right)^n = e^5$. (2) $\lim_{n \rightarrow \infty} \left(1 - \frac{1}{n}\right)^n = e^{-1}$.

★ As for (2), $\frac{1}{e}$ is equally acceptable.

(3) $\lim_{n \rightarrow \infty} \left(1 + \frac{\sqrt{3}}{n}\right)^n = e^{\sqrt{3}}$.

[VII] (3pts) Agree

$$e^2 = 1 + \frac{1}{1!} \cdot 2 + \frac{1}{2!} \cdot 2^2 + \frac{1}{3!} \cdot 2^3 + \frac{1}{4!} \cdot 2^4 + \frac{1}{5!} \cdot 2^5 + \dots$$

Mimic this and write out the following quantity exactly in this format.

$$\sqrt[4]{e}.$$

[**Answer**]:

$$\sqrt[4]{e} = 1 + \frac{1}{1!} \cdot \left(\frac{1}{4}\right) + \frac{1}{2!} \cdot \left(\frac{1}{4}\right)^2 + \frac{1}{3!} \cdot \left(\frac{1}{4}\right)^3 + \frac{1}{4!} \cdot \left(\frac{1}{4}\right)^4 + \frac{1}{5!} \cdot \left(\frac{1}{4}\right)^5 + \dots$$