

**Math 105 TOPICS IN MATHEMATICS**  
**SOLUTION FOR REGULAR HOMEWORK – VII (03/02)**

March 4 (Wed), 2015

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[I] (4pts) You started a lemonade business. Words spread fast and the sales dramatically increased (Table 1 below):

day	\$
1	1
2	2
3	6
4	24
5	120
6	720
⋮	⋮

On Day  $n$ , your sales will be  $n$  times the previous day sales. Suppose the same patterns hold until Day 75. Then the sales on Day 75 is how much? The answer involves ‘!’ (the factorial symbol). You don’t have to simplify (calculate) the answer.

[ Answer ]: \$ 75!.

[II] (4pts) Complete the identities:

$$(1) \quad \binom{8}{3} = \frac{8 \cdot 7 \cdot 6}{\boxed{3}!} \quad (2) \quad \binom{11}{4} = \frac{\boxed{11} \cdot \boxed{10} \cdot \boxed{9} \cdot \boxed{8}}{4!}.$$

[III] (8pts) (1) Which one is bigger:  $\left(1 + \frac{1}{27}\right)^{27}$  or  $\left(1 + \frac{1}{28}\right)^{28}$  ?

[Answer]:  $\left(1 + \frac{1}{28}\right)^{28}$  is bigger.

(2) Which one is bigger:  $\left(1 + \frac{1}{10^{100}}\right)^{10^{100}}$  or 2.5?

[Answer]:  $\left(1 + \frac{1}{10^{100}}\right)^{10^{100}}$  is bigger.

(3) True or false :  $\left(1 + \frac{1}{4}\right)^4 < 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!}$ .

[Answer]: True. (This is clear if you binomially expand it.)

(4) True or false : If you choose a large enough  $n$  then

$$\left(1 + \frac{1}{n}\right)^n > 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \frac{1}{5!} + \frac{1}{6!}.$$

[Answer]: True. (“Review of Lectures – XII”, page 17, ‘Fact B’.)

[IV] (6pts) (1)  $\sqrt{49} = 7$ . (2)  $\sqrt{100} = 10$ . (3)  $\sqrt{256} = 16$ .

[V] (8pts)

(1a) Suppose a number  $r$  has a decimal expression that stops after finitely many digits under the decimal point. Is  $r$  a rational number?

[Answer]: Yes.

(1b) Give an example of such  $r$ .

[Example]:  $\frac{1}{2} = 0.5$ .

(2a) Suppose a number  $r$  has the following decimal expression:

$$0.\underline{027}\underline{027}\underline{027}\underline{027}\underline{027}\dots$$

where the 'unit' 027 repeats permanently. Is this a rational number?

[Answer]: Yes.

(2b) Write the number in (2a) as an integer divided by another integer.

$$[\text{Answer}]: \frac{27}{999} = \frac{1}{37}.$$