Math 105 TOPICS IN MATHEMATICS

SOLUTION FOR REGULAR HOMEWORK – VI (02/20)

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[I] (6pts)   (1) $3 \cdot x \cdot 5 = 15\,x.$ (2) $35 \cdot x^4 \cdot 2^3 = 280\,x^4.$

[II] (3pts)   The outcome of substituting $a = -8$ in $\left(x+a\right)^4$ is

$$\left(x-8\right)^4.$$

[III] (6pts)   (1) $1 - \dfrac{1}{10} = \dfrac{9}{10}.$

(2) $\dfrac{1}{a} \cdot \dfrac{1}{b} \cdot \dfrac{1}{c} \cdot \dfrac{1}{d} = \dfrac{1}{\boxed{abcd}}$.

[IV] (9pts)   (1)   <u>True or false</u> :

  $\underline{\underline{\text{If}}}$ $\boxed{a < b}$ $\underline{\underline{\text{and}}}$ $\boxed{t > 0}$ $\underline{\underline{\text{then}}}$ $\boxed{t\,a < t\,b}$.

$\left[\underline{\text{Answer}}\right]$: True.

(2)   <u>True or false</u> : $\underline{\underline{\text{If}}}$ $\boxed{c > d}$ $\underline{\underline{\text{then}}}$ $\boxed{1 - c > 1 - d}$.

$\left[\underline{\text{Answer}}\right]$: False.

(3)   Which number is bigger, $\dfrac{9}{9} \cdot \dfrac{8}{9} \cdot \dfrac{7}{9} \cdot \dfrac{6}{9}$ or $\dfrac{10}{10} \cdot \dfrac{9}{10} \cdot \dfrac{8}{10} \cdot \dfrac{7}{10}$?

$\boxed{\text{Answer}}$: $\dfrac{10}{10} \cdot \dfrac{9}{10} \cdot \dfrac{8}{10} \cdot \dfrac{7}{10}$ is bigger.

$\boxed{\text{Reason}}$:   Clearly

$$\frac{9}{9} = \frac{10}{10}, \qquad \frac{8}{9} < \frac{9}{10}, \qquad \frac{7}{9} < \frac{8}{10} \quad \text{and} \quad \frac{6}{9} < \frac{7}{10}.$$

[V] (6pts)   (1) $\dfrac{1}{2} + \dfrac{1}{4} + \dfrac{1}{8} + \dfrac{1}{16} + \dfrac{1}{32} + \dfrac{1}{64} = 1 - \dfrac{1}{\boxed{64}}.$

(2)   Let n be a poistive integer. Then

$$\frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4} + \cdots + \frac{1}{2^n} = 1 - \frac{1}{2^{\boxed{n}}}.$$