Math 105 TOPICS IN MATHEMATICS

SOLUTION FOR REGULAR HOMEWORK – V (02/09)

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[I] (6pts) (1) $0^{12} = 0$. (2) $1^{24} = 1$. (3) $\Big(-1\Big)^{99} = -1$.

[II] (6pts) (1) $\dbinom{11}{4} = \dfrac{11 \cdot 10 \cdot 9 \cdot 8}{1 \cdot 2 \cdot 3 \cdot 4}$.

(2) $\dbinom{24}{7} = \dfrac{24 \cdot 23 \cdot 22 \cdot 21 \cdot 20 \cdot 19 \cdot 18}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7}$.

(3) $\dbinom{100}{12} = \dfrac{100 \cdot 99 \cdot 98 \cdot 97 \cdot 96 \cdot 95 \cdot 94 \cdot 93 \cdot 92 \cdot 91 \cdot 90 \cdot 89}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8 \cdot 9 \cdot 10 \cdot 11 \cdot 12}$.

[III] (8pts) (a) $\Big(x + y\Big)^5$

$$= \binom{5}{0} x^5 + \binom{5}{1} x^4 y + \binom{5}{2} x^3 y^2 + \binom{5}{3} x^2 y^3 + \binom{5}{4} x y^4 + \binom{5}{5} y^5$$

$$= x^5 + 5\, x^4 y + 10\, x^3 y^2 + 10\, x^2 y^3 + 5\, x y^4 + y^5,$$

(b) $\Big(x + y\Big)^6$

$$= \binom{6}{0} x^6 + \binom{6}{1} x^5 y + \binom{6}{2} x^4 y^2 + \binom{6}{3} x^3 y^3 + \binom{6}{4} x^2 y^4$$
$$+ \binom{6}{5} x y^5 + \binom{6}{6} y^6$$

$$= x^6 + 6\, x^5 y + 15\, x^4 y^2 + 20\, x^3 y^3 + 15\, x^2 y^4 + 6\, x y^5 + y^6.$$

[IV] (10pts)

(1) $2^{11} - 1$ is written as the product of two primes. One of the two primes is 23. What is the other prime?

[Answer]: 89.

[Work]:

$$\begin{array}{r|l} & 89 \\ \hline 23 & 2047 \\ & 184 \\ \hline & 207 \\ & 207 \\ \hline & 0 \end{array}$$

(2) True or false : "*If* n *is a prime, then* $2^n - 1$ *is a prime.*"

[Answer]: False. (Part (1) $\left(2^{11} - 1 = 23 \cdot 89\right)$ serves as a counterexample.)

(3) True or false : "*If* $2^n - 1$ *is a prime, then* n *is a prime.*"

[Answer]: True.

(4) True or false : "*If* $2^n + 1$ *is a prime, then* n *is a 2-to-the-power.*"

[Answer]: True.

(5) Is $2^{32} + 1$ a prime? If not, what is the smallest prime that divides $2^{32} + 1$?

[Answer]: 641.