# Math 105 TOPICS IN MATHEMATICS
## REGULAR HOMEWORK – XI

April 29 (Wed), 2015

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**Line #:** 52920.

⋆ **Due date:** **Monday, May 4th, 2015** .

⋆ **Your paper will be collected in class. No late homework will be accepted. Please see " Rules, Policies and Protocols " p.14 about homework policy.**

[I] (16pts) Evaluate

(1) $\sin 0$. (2) $\sin \dfrac{\pi}{6}$. (3) $\cos \left(-\dfrac{\pi}{4}\right)$. (4) $\sin \dfrac{\pi}{3}$.

(5) $\cos \dfrac{\pi}{2}$. (6) $\cos \dfrac{5\pi}{6}$. (7) $\sin \left(-\dfrac{\pi}{2}\right)$. (8) $\cos \pi$.

[II] (4pts)

(1) Find the distance $\left|PQ\right|$ between $P = \left(0,\ 0\right)$ and $Q = \left(5,\ 6\right)$.

(2) Find the distance $\left|PQ\right|$ between $P = \left(-4,\ 0\right)$, and $Q = \left(3,\ 1\right)$.

[III] (4pts) (1) What number does $\left(\cos\theta\right)^2 + \left(\sin\theta\right)^2$ equal?

(2) Paraphrase (1):

" The distance between

$$P = \left( \boxed{\phantom{xxxxxx}} , \boxed{\phantom{xxxxxx}} \right)$$

and the coordinate origin $O = \left(0,\ 0\right)$ is always 1. "

[IV] (3pts) Let

$$P = \left(\cos\frac{2\pi}{5},\ \sin\frac{2\pi}{5}\right), \qquad Q = \left(\cos\frac{3\pi}{5},\ \sin\frac{3\pi}{5}\right),$$

$$R = \left(\cos\frac{\pi}{5},\ \sin\frac{\pi}{5}\right), \qquad S = \left(1,\ 0\right).$$

True or false : "$|PQ|$ and $|RS|$ are equal."

Explain.

[V] (5pts) Assume

$$B_8{}^\circ(x) = x^8 - 4x^7 + \frac{14}{3}x^6 - \frac{7}{3}x^4 + \frac{2}{3}x^2,$$

and recover $B_8$ and $B_9{}^\circ(x)$. Follow the steps below:

(1) Add $B_8$ to $B_8{}^\circ(x)$. So, write out

$$B_8(x) = x^8 - 4x^7 + \frac{14}{3}x^6 - \frac{7}{3}x^4 + \frac{2}{3}x^2 + B_8.$$

(2) Take its antiderivative. So, complete the following line:

$$\int B_8(x)\,dx = \frac{1}{9}x^9 - \frac{1}{2}x^8 + \underline{\hspace{6cm}} + B_8 \cdot x + C.$$

This is $\frac{1}{9}B_9{}^\circ(x)$. So

$$\frac{1}{9}B_9{}^\circ(x) = \frac{1}{9}x^9 - \frac{1}{2}x^8 + \underline{\hspace{6cm}} + B_8 \cdot x + C.$$

(3) Substitute $x = 0$ and $x = 1$ into (2) independently. The outcomes are both 0. Thus

$$\begin{cases} 0 = \frac{1}{9}0^9 - \frac{1}{2}0^8 + \underline{\hspace{12em}} + B_8 \cdot 0 + C. \\ 0 = \frac{1}{9}1^9 - \frac{1}{2}1^8 + \underline{\hspace{12em}} + B_8 \cdot 1 + C. \end{cases}$$

The first of the two equations reads $0 = C$. So $C = 0$. Taking this into account, the second of the two equations becomes

$$0 = \frac{1}{9} - \frac{1}{2} + \underline{\hspace{12em}} + B_8.$$

Solve it for $B_8$.

$$B_8 = \underline{\hspace{5em}}.$$

(4) Substitute the value for $B_8$ which you found in (3) into the result of (2). Multiply 9 to the both sides. So

$$B_9{}^{\circ}(x) = x^9 - \frac{9}{2}x^8 + \underline{\hspace{18em}}.$$