Math 105 TOPICS IN MATHEMATICS
SOLUTION FOR REGULAR HOMEWORK – X (04/17)

April 20 (Mon), 2015

Instructor: Yasuyuki Kachi

   Line #:   52920.

[I] (6pts)   (1) $\dfrac{d}{dx} \, x^4 \; = \; 4x^3.$ (2) $\dfrac{d}{dx} \, x^{20} \; = \; 20x^{19}.$

(3) $\dfrac{d}{dx} \left(\dfrac{1}{11} x^{11} \right) \; = \; x^{10}.$

[II] (6pts)   (1) $\dfrac{d}{dx} \left(x^6 \, + \, 2x^3 \right) \; = \; 6x^5 \, + \, 6x^2.$

(2) $\dfrac{d}{dx} \left(x^7 \, - \, 3x^4 \, + \, x \right) \; = \; 7x^6 \, - \, 12x^3 \, + \, 1.$

(3) $\dfrac{d}{dx} \left(\dfrac{1}{4} x^4 \, - \, \dfrac{1}{2} x^3 \, + \, \dfrac{1}{6} x^2 \, - \, \dfrac{1}{30} \right) \; = \; x^3 \, - \, \dfrac{3}{2} x^2 \, + \, \dfrac{1}{3} x.$

[III] (4pts)

(1) $\dfrac{d}{dx} \, \dfrac{1}{9!} \, x^9 \; = \; \dfrac{1}{8!} \, x^8.$ (2) $\dfrac{d}{dx} \, \dfrac{1}{40!} \, x^{40} \; = \; \dfrac{1}{39!} \, x^{39}.$

[IV] (5pts)

$$\dfrac{d}{dx} \left(1 + \dfrac{1}{1!} x + \dfrac{1}{2!} x^2 + \dfrac{1}{3!} x^3 + \dfrac{1}{4!} x^4 + \dfrac{1}{5!} x^5 + \dfrac{1}{6!} x^6 + \dfrac{1}{7!} x^7 + \dfrac{1}{8!} x^8 + \dfrac{1}{9!} x^9 \right)$$

$$= \; 1 + \dfrac{1}{1!} x + \dfrac{1}{2!} x^2 + \dfrac{1}{3!} x^3 + \dfrac{1}{4!} x^4 + \dfrac{1}{5!} x^5 + \dfrac{1}{6!} x^6 + \dfrac{1}{7!} x^7 + \dfrac{1}{8!} x^8.$$

[V] (9pts)   Recall

$$1^2 + 2^2 + 3^2 + 4^2 + \cdots + n^2 \quad = \quad \frac{1}{3}n^3 + \frac{1}{2}n^2 + \frac{1}{6}n$$

.

Give similar formulas for

(1) $1^3 + 2^3 + 3^3 + 4^3 + \cdots + n^3$,

(2) $1^4 + 2^4 + 3^4 + 4^4 + \cdots + n^4$,   and

(3) $1^5 + 2^5 + 3^5 + 4^5 + \cdots + n^5$.

No work necessary.

$\left[\,\textbf{Answers}\,\right]$:

(1) $1^3 + 2^3 + 3^3 + 4^3 + \cdots + n^3 \quad = \quad \dfrac{1}{4}n^4 + \dfrac{1}{2}n^3 + \dfrac{1}{4}n^2$,

(2) $1^4 + 2^4 + 3^4 + 4^4 + \cdots + n^4 \quad = \quad \dfrac{1}{5}n^5 + \dfrac{1}{2}n^4 + \dfrac{1}{3}n^3 - \dfrac{1}{30}n$,

(3) $1^5 + 2^5 + 3^5 + 4^5 + \cdots + n^5 \quad = \quad \dfrac{1}{6}n^6 + \dfrac{1}{2}n^5 + \dfrac{5}{12}n^4 - \dfrac{1}{12}n^2$.