Math 105 TOPICS IN MATHEMATICS

SOLUTION FOR REGULAR HOMEWORK – I (01/23)

January 28 (Wed), 2015

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Line #: 52920.

[I] (10pts)

(a) 17 is a prime. (b) 25 is not a prime. $\left(\text{Indeed, } 25 = 5 \cdot 5.\right)$

(c) 31 is a prime. (d) 87 is not a prime. $\left(\text{Indeed, } 87 = 3 \cdot 29.\right)$

(e) 101 is a prime. (Indeed, 101 is not divisible by either one of 2, 3, 5 or 7, the only primes whose square is less than 101.)

[II] (9pts) True or false:

(1) "There are infinitely many prime numbers."

— The answer is "true".

(2) "There are 1000000000000 (one trillion) consecutive positive integers none of which is a prime."

— The answer is "true".

(3) "No matter how large a number you choose, there are two primes above that number and whose gap is less than 70000000 (seventy million)."

— The answer is "true".

$\star$ [Sidenote] The statement (3) is a theorem by Dr. Yitang Zhang (2013).

[III] (3pts) Identify the only <u>even</u> prime number.

— The answer is 2.

[IV] (4pts) (1) The Riemann Hypothesis was proposed by Bernhart Riemann.

[Sidenote] That was in 1859.

(2) Has it been solved, as of January 23, 2015? (Answer 'Yes' or 'No'.)

— The answer is 'no'. It is still an open problem as of today.

[V] (4pts)

(a)
$$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12 + 13 + 14 + 15 + 16 + 17 + 18 + 19 + 20 + 21 + 22 + 23$$

is found as the substitution of $n = 23$ in $\frac{1}{2}n\left(n+1\right)$. It is performed as

$$\frac{1}{2} \cdot 23 \cdot 24 = 276.$$

(b)
$$1 + 2 + 3 + 4 + 5 + \cdots + 1000$$

is found as the substitution of $n = 1000$ in $\frac{1}{2}n\left(n+1\right)$. It is performed as

$$\frac{1}{2} \cdot 1000 \cdot 1001 = 500500.$$