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## Math 105 TOPICS IN MATHEMATICS MIDTERM EXAM – I (Take-home)

March 6 (Fri), 2015

Instructor: Yasuyuki Kachi

Line #: 52920.

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This take-home part of Midterm Exam is worth 60 points and <u>is</u> <u>due in class Wednesday, March 11th, 2015.</u> Submission after 1:00 pm, March 11th will not be accepted.

• Be sure to write your answers neatly, precisely, and with complete sentences. You may use notes and handed out materials, but no outside help.

## • Print off one entire set of this exam. Write answers in the printed sheets. You may not supply your own (blank) sheet.

 $\star$  In problem [I] below we work on a model where one can divide any dollar amount by any large number (integer). Also, we never round figures. So, one-third of a dollar is never the same as 33 cents (because 33 cents is one-third of 99 cents).

[I] (Take-home; 20pts) You open a bank account, deposit a dollar in that account.

(1) Your bank offers 100 percent interest annually.

After one year, your balance is \$

(2) Suppose your bank offers a compound interest with 100 percent rate annually.

(2a) After two years, your balance is \$ .

(2b) After three years, your balance is \$

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([I] continued)

(3) Suppose the compounding takes place five times annually. So every  $\frac{1}{5}$ -th of a year  $\frac{1}{5} \cdot 100 = 20$  percent of your balance will be accrued as an interest.

(3b) After the 
$$\frac{2}{5}$$
-th of a year, your balance is  $\$$ 

(3c) After the one year, your balance is 
$$\$ \left( \begin{array}{c} 1 + \frac{1}{5} \end{array} \right)$$

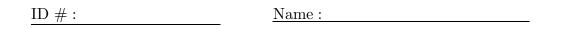
(4) Suppose the compounding takes place  $10^{100}$  times annually. So every  $\frac{1}{10^{100}}$ -th of a year,  $\frac{1}{10^{100}}$  times 100 percent of your balance will be accrued as an interest.

After one year, your balance is 
$$\begin{pmatrix} 1 + \frac{1}{2} \end{pmatrix}$$
.

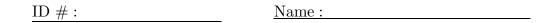
(5) Is your answer in (4)

- $\Box$  between \$1 and \$2.  $\Box$  between \$2 and \$2.50.
- $\Box$  between \$2.50 and \$3.00.  $\Box$
- $\Box$  more than \$3.

(<u>Check one.</u>)



[II] (Take-home; 20pts) (a) <u>Use calculator</u> to pull the decimal expressions of the numbers in each of (a5) through (a10).



([II] continued)

(b) <u>Use calculator</u> to find the smallest positive integer n such that

$$\left(1+\frac{1}{n}\right)^n$$

is bigger than the value in (a4) above (= 2.7083333...).

n =\_\_\_\_\_. 4

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([II] continued)

(c) True or false :

"Let k be an arbitrarily chosen positive integer, and fixed. If you choose a large enough n, then

$$\left(1 + \frac{1}{n}\right)^{n} > 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \frac{1}{5!} + \dots + \frac{1}{k!}.$$
"  

$$\Box \quad \text{True.} \qquad \Box \quad \text{False.} \qquad \left(\underline{\text{Check one.}}\right)$$

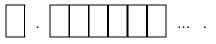
(d1) Give one definition of e.

$$e = \lim_{n \to \infty} \left( 1 + \bigsqcup \right)^n .$$

(d2) Give another definition of e.

$$e = \lim_{n \to \infty} \left( 1 + \frac{1}{\Box} + \frac{1}{\Box} + \frac{1}{\Box} + \frac{1}{\Box} + \frac{1}{\Box} + \dots + \frac{1}{\Box} \right).$$

(d3) The decimal expression of e up to the first six place under the decimal point



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[III] (Take-home; 20pts) Prove that  $\sqrt{3}$  is an irrational number.

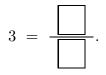
**Proof.** Proof by contradiction. Suppose  $\sqrt{3}$  is written as

$$\sqrt{3} = \frac{k}{m}$$

using some integers k and m (where  $m \neq 0$ ).

First, if both k and m are divisible by 3, then we may simultaneously divide both the numerator and the denominator by 3 (and the value of the fraction stays the same). After that procedure, suppose both the numerator and the denominator still remain to be divisible by \_\_\_\_\_\_, then we repeat the same procedure as many times as necessary until at least one of the numerator and the denominator is not divisible by \_\_\_\_\_\_. Thus we may assume, without loss of generality, that at least one of k and m is

Under this assumption, square the both sides of the identity  $\sqrt{3} = \frac{k}{m}$ , thus



This is the same as

$$= k^2.$$

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([III] continued)	
The left-hand side of this last	identity is clearly divisible by, so this last
identity forces its right-hand side	de to be
That in turn implies $k$ is	because if $k$ is,
	then $k^2$ is
But then $k$ being divisible by	implies $k^2$ is
So by virtue of the above last id	dentity $3 m^2$ is divisible by or the same to say,
$m^2$ is divisible by $\square$ . This	is implies that $m$ is
In short, both $k$ and $m$ are	
This contradicts our assumpt	tion. The proof is complete. $\Box$

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