Your TA: _____ Usual Seat #: ☐ – ☐

Exam Seat #: ☐ – ☐

Math 105 TOPICS IN MATHEMATICS

MIDTERM EXAM – I (In-class)

March 11 (Wed), 2015

Instructor: Yasuyuki Kachi

Line #: 52920.

ID # : _____ Name : _____

This "in-class" portion of the exam is worth 60 points, plus extra 10 points. Note that there is a "take-home" portion, which is worth 60 points. The duration of this "in-class" portion of the exam is 50 minutes (start at 1:00pm, finish at 1:50pm).

[I] (18pts) (1) Are there infinitely many, or finitely many, prime numbers?

☐ Infinitely many. ☐ Finitely many. (Check one.)

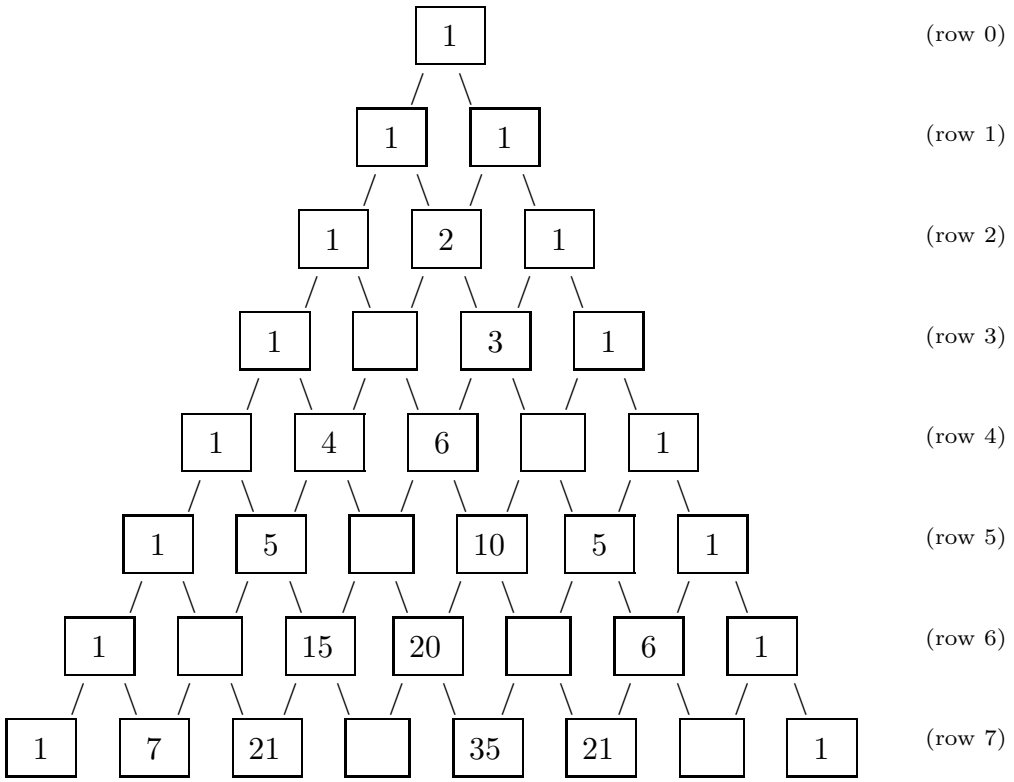
(2) Circle all the prime numbers between 2 and 50.

	2,	3,	4,	5,	6,	7,	8,	9,	10,
11,	12,	13,	14,	15,	16,	17,	18,	19,	20,
21,	22,	23,	24,	25,	26,	27,	28,	29,	30,
31,	32,	33,	34,	35,	36,	37,	38,	39,	40,
41,	42,	43,	44,	45,	46,	47,	48,	49,	50.

Line #: 52920.

ID # : _____ Name : _____

[II] (7pts) Fill in the empty boxes in the Pascal's triangle (there are seven of them):



[III] (3pts) Complete the identities:

(1) $\binom{9}{3} = \dfrac{9 \cdot 8 \cdot 7}{\square\,!}$. (2) $\binom{15}{4} = \dfrac{\boxed{15} \cdot \square \cdot \square \cdot \boxed{12}}{4\,!}$.

Line #: 52920.

ID # : _____ Name : _____

[IV] (4pts) You started a lemonade business. Words spread fast and the sales dramatically increased (Table 1 below):

day	$
1	1
2	2
3	6
4	24
5	120
6	720
⋮	⋮

On Day n, your sales will be n times the previous day sales. Suppose the same patterns hold until Day 90. Then the sales on Day 90 is how much? The answer involves '!' (the factorial symbol). You don't have to simplify (calculate) the answer.

_____ .

[V] (7pts) Circle all 2-to-the-powers among the numbers listed below.

4, 6, 8, 10, 12, 16, 24, 32, 36, 48,

54, 64, 72, 84, 96, 108, 128, 144, 216, 256.

Line #: 52920.

ID # : _____ Name : _____

[VI] (8pts) Simplify:

(1) $\sqrt[4]{16} =$ _____ . (2) $\sqrt{7} \cdot \sqrt{5} =$ _____ .

(3) $\sqrt{\sqrt{3}} =$ _____ . (4) $100^0 =$ _____ .

[VII] (3pts) Fill concrete numbers in the boxes:

(1) $\Big(x+y\Big)^4 = x^4 + \boxed{4}\, x^3\, y + \boxed{}\, x^2\, y^2 + \boxed{}\, x\, y^3 + y^4$.

(2) If $a + b + c = 0$ then $a^3 + b^3 + c^3 = \boxed{}\, a\, b\, c.$

[VIII] (4pts)

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \frac{1}{64} + \frac{1}{128} =$$ _____ .

Line #: 52920.

ID # : _____ Name : _____

[IX] (6pts)

(1) $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10$

$=$ _____ .

(2)
$$\begin{aligned}
&1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 \\
&+ 11 + 12 + 13 + 14 + 15 + 16 + 17 + 18 + 19 + 20 \\
&+ 21 + 22 + 23 + 24 + 25 + 26 + 27 + 28 + 29 + 30 \\
&+ 31 + 32 + 33 + 34 + 35 + 36 + 37 + 38 + 39 + 40 \\
&+ 41 + 42 + 43 + 44 + 45 + 46 + 47 + 48 + 49 + 50 \\
&+ 51 + 52 + 53 + 54 + 55 + 56 + 57 + 58 + 59 + 60 \\
&+ 61 + 62 + 63 + 64 + 65 + 66 + 67 + 68 + 69 + 70 \\
&+ 71 + 72 + 73 + 74 + 75 + 76 + 77 + 78 + 79 + 80 \\
&+ 81 + 82 + 83 + 84 + 85 + 86 + 87 + 88 + 89 + 90 \\
&+ 91 + 92 + 93 + 94 + 95 + 96 + 97 + 98 + 99 + 100
\end{aligned}$$

$=$ _____ .

[Hint for (2)] :

Add up 1 and 100, then add up 2 and 99, then add up 3 and 98, *etc.*

Line #: 52920.

ID # : ____________________ Name : ____________________________

[X] (Extra 3pts) Babylonians knew that

$$1 + \frac{24}{60} + \frac{51}{60^2} + \frac{10}{60^3} = \frac{30547}{21600}$$

gives a good approximation of $\sqrt{2}$. Indeed,

$$\text{(1)} \qquad \frac{30547}{21600} = 1.41421296296296\ldots ,$$

$$\text{(2)} \qquad \sqrt{2} = 1.41421356237309\ldots .$$

The difference between line (1) and line (2) is less than $\frac{1}{60^3}$.

Now, fill in a concrete integer between 0 and 59 into the box below, so that the difference between $\sqrt{2}$ and line (3) below does not exceed $\frac{1}{60^4}$:

$$\text{(3)} \qquad 1 + \frac{24}{60} + \frac{51}{60^2} + \frac{10}{60^3} + \frac{\square}{60^4}.$$

[**Hint**] **:** Round-down the outcome of the following number (at the decimal point):

(	√	(	2	)	−	(	1	+	2	4	/	6	0	+	5	1	/	(	6	0	^
2	)	+	1	0	/	(	6	0	^	3	)	)	)	*	(	6	0	^	4	)	ENTER

Line #: 52920.

ID # : _____ Name : _____

[XI] (Extra 7pts) In the formula

$$1\cdot 2 + 2\cdot 3 + 3\cdot 4 + \cdots + n\cdot\big(n+1\big) \;=\; \frac{1}{3}n\big(n+1\big)\big(n+2\big)$$

$\Big($essentially covered in class$\Big)$, substitute $n = 9$:

(1) $1\cdot 2 + 2\cdot 3 + 3\cdot 4 + 4\cdot 5 + 5\cdot 6 + 6\cdot 7 + 7\cdot 8 + 8\cdot 9 + 9\cdot 10 \;=\; 330.$

Meanwhile,

(2) $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 \;=\; 45.$

Subtract line (2) from line (1), and thereby evaluate

$$1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2 + 8^2 + 9^2 \;=\; \_\_\_\_\_\,.$$

Work.

$$\begin{array}{l}
\underbrace{1\cdot 2} + \underbrace{2\cdot 3} + \underbrace{3\cdot 4} + \underbrace{4\cdot 5} + \underbrace{5\cdot 6} + \underbrace{6\cdot 7} + \underbrace{7\cdot 8} + \underbrace{8\cdot 9} + \underbrace{9\cdot 10} \\
\quad\|\qquad\|\qquad\|\qquad\|\qquad\|\qquad\|\qquad\|\qquad\|\qquad\| \\
\boxed{2} + \boxed{6} + \boxed{12} + \boxed{20} + \boxed{30} + \boxed{42} + \boxed{56} + \boxed{72} + \boxed{90} \;=\; \boxed{330} \\
\boxed{1} + \boxed{2} + \boxed{3} + \boxed{4} + \boxed{5} + \boxed{6} + \boxed{7} + \boxed{8} + \boxed{9} \;=\; \boxed{45} \\
-) \;\text{---} \\
\boxed{1} + \boxed{4} + \boxed{9} + \boxed{} + \boxed{} + \boxed{} + \boxed{} + \boxed{} + \boxed{} \;=\; \boxed{}
\end{array}$$